Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Problem Solving-I

Subject Code: 5SC02PRS1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 20/10/2018** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a. Evaluate $\int_C \frac{1}{(z-2)(z-3)}\, dz$, where C is the circle $|z| = 1$. **(01)**

b. Show that $v_1 = (-1, 2, 4)$ and $v_2 = (5, -10, -20)$ forms a linearly dependent set in R^3. **(01)**

c. Find the Laurent's series of the function $f(z) = \frac{1}{z^2(1-z)}$ about $z = 0$. **(01)**

d. Find modulus of $\frac{1+i}{1-\sqrt{3}i}$. **(01)**

e. $W = \{(x, y, z) |\ y = x + z + 1\}$ is subspace of R^3. (True/False) **(01)**

f. An analytic function with constant modulus is constant. (True/False) **(01)**

g. $f(z) = \frac{1}{z-1}$ has singular point at $z = 1$. (True/False) **(01)**

Q-2 **Attempt all questions** **(14)**

a. Evaluate $\int_C \frac{e^{az}}{z^2+1}\, dz$, where C is the circle $|z| = 2$, where a is measured in radians. **(05)**

b. Construct the analytic function $f(z) = u + iv$ where $v = e^x(x \sin y + y \cos y)$. **(05)**

c. Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i$ and $z_3 = -2$ into the points $w_1 = 1, w_2 = i$ and $w_3 = -1$ respectively. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a. Determine the order of each pole and calculate residue at each of the pole of $f(z) = \frac{1-2z}{z(z-1)(z-2)}$. **(05)**

b. Find z if $\arg(z + 2i) = \frac{\pi}{4}$ and $\arg(z - 2i) = \frac{3\pi}{4}$. **(05)**

c. Evaluate $\int_C (z^2 + 3z)\, dz$ along the circle $|z| = 2$ from $(2, 0)$ to $(0, 2)$. **(04)**

Q-3 **Attempt all questions** **(14)**

a. Obtain the Laurent's series which represents the function $f(z) = \frac{1}{z(z^2-3z+2)}$ for the regions $(i)\ 0 < |z| < 1,\ \ (ii)\ 1 < |z| < 2,\ (iii) |z| > 2$. **(07)**

b. Express the polynomial $p(x) = -9 - 7x - 15x^2$ as a linear combination of $p_1(x) = 2 + x + 4x^2, p_2(x) = 1 - x + 3x^2, p_3(x) = 3 + 2x + 5x^2$. **(04)**

c. Show that $W = \{(x, y)|x = 3y\}$ is subspace of R^2. **(03)**

OR

Q-3 **Attempt all questions** **(14)**

a. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **(07)**

b. Prove that $\tan^{-1} z = \frac{i}{2} \log\left(\frac{i+z}{i-z}\right)$. **(04)**

c. Verify the Cauchy-Riemann equations for the function $f(z) = \frac{1}{z}$. **(03)**

SECTION – II

Q-4 **Attempt the following questions** **(07)**

a. Find particular integral of $(D^2 + 3D + 2)y = e^{2x}$. **(01)**

b. Evaluate: $\begin{vmatrix} -3 & 0 & 0 \\ 6 & 4 & 0 \\ -1 & 2 & 5 \end{vmatrix}$. **(01)**

c. Solve: $(D^2 + 6D + 9)y = 0$. **(01)**

d. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ satisfies the matrix equation $A^2 - kA + 2I = 0$, then what is the value of k? **(01)**

e. If $V = \{a_1, a_2, \dots, a_{100} : a_1 + a_2 = 0, a_3 + a_4 = 0\}$, then find $\dim V$. **(01)**

f. $y = A\cos x + B\sin x$ is solution of $\frac{d^2y}{dx^2} + y = 0$. (True/False) **(01)**

g. The function $f(x, y) = 4x^2 + y^2$ on $R: |x| \le 1, |y| \le 1$ satisfies Lipschitz condition. (True/False) **(01)**

Q-5 **Attempt all questions** **(14)**

a. Find the rank of matrix by normal form, where $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. **(05)**

b. Solve the following simultaneous equations **(05)**

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0, \frac{dy}{dt} + 5x + 3y = 0.$$

c. Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. **(04)**

OR

Q-5 **Attempt all questions** **(14)**

a. Solve: $(D^2 - 5D + 6)y = e^x \cos 2x$. **(05)**

b. Apply method of variation of parameters to solve $y'' - 2y' = e^x \sin x$. **(05)**

c. Write the matrix A of the quadratic form $6x^2 + 35y^2 + 11z^2 + 4zx$. Find the **(04)** eigenvalues of A.

Q-6		**Attempt all questions**	**(14)**
	a.	Find the characteristic equation of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$. Hence find A^{-1}.	**(07)**
	b.	Find particular integral of $(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$.	**(04)**
	c.	Solve: $x(x-1)\frac{dy}{dx} - (x-2)y = x^2(2x-1)$.	**(03)**

OR

Q-6		**Attempt all questions**	**(14)**
	a.	Find the eigenvalues and the corresponding eigenfunctions of $y'' + \lambda y = 0, y(0) = 0 = y'(L)$.	**(07)**
	b.	Determine whether the given vectors $v_1 = (1,-1,1), v_2 = (0,1,2), v_3 = (3,0,-1)$ forms basis for R^3.	**(04)**
	c.	Let T be a transformation from R^3 into R^1 defined by $T(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$. Show that T is not a linear transformation.	**(03)**